

### Contents

- 1. Introduction
- 2. Delete Relaxation Heuristics
- 3. Critical Path Heuristics
- 4. Context-Enhanced Additive Heuristic
- 5. Landmark Heuristics
- 6. Abstraction Heuristics
- 7. Final Comments

We try to give a general overview with the basic ideas and intuitions for different heuristics

See references for details

Planning as Search			
Heuristics			
Problem Representation			
Emil Keyder, Silvia Richter	Heuristics: 1. Introduction	June 2011	2 / 24

## Planning as Search Planning as Heuristic Search Successful and robust • Top four planners in the satisficing track of IPC6 • Several top planners in the optimal track • Many well-performing planners from previous competitions Standardized framework • Mix and match heuristics and search techniques



### Solutions are plans

Solutions of minimum cost are optimal plans

Heuristics help by:

graph

Heuristics: 1. Introduction

▶ Starting from  $v_0$ , explore reachable vertices until  $v_{g} \in G$  is found

• Delaying or ruling out the exploration of unpromising regions of the

• Guiding the search towards promising regions of the graph

A solution is a sequence of edges  $\pi = \langle e_0, \ldots, e_n \rangle$  that forms a path from  $v_0$  to some  $v_g \in V_G$ 

An optimal solution is a path with minimum total cost, where the cost of a path is given by the sum of its edge costs:

 $\mathit{cost}(\pi) = \sum_{e \in \pi} \mathit{cost}(e)$ 

Emil Keyder, Silvia Richter

Solving Search Problems

Uniform-cost search, Dijkstra

Heuristics: 1. Introduction

Planning as Search

Brute-force approach: Systematically explore full graph

June 2011 6 / 24

### Heuristics

### Heuristics: What are they?

Heuristics are functions  $h: V \mapsto \mathbb{R}^+_0$  that estimate cost of path to a goal node

### Definition

 $h^*(v)$  is the cost of the lowest-cost path from v to some  $v' \in V_G$ 

### $h^*(v) \rightarrow \text{optimal solution in linear time}$

 $\rightsquigarrow$  Intractable to compute in general, but useful comparison point

Objective : get as close as possible to  $h^*$ 

Emil Keyder, Silvia Richter

Heuristics: 1. Introduction

Heuristics

Maximum of Admissible Heuristics

Maximum of admissible heuristics

Let  $h_1$ ,  $h_2$  be two admissible heuristics. Note that

 $h_1(s) \leq h^*(s) \wedge h_2(s) \leq h^*(s) \implies \max(h_1(s), h_2(s)) \leq h^*(s)$ 

The maximum of two admissible heuristics is a more informed admissible heuristic



### Sum of Admissible Heuristics

Sum of two admissible heuristics not in general admissible  $\rightsquigarrow h^* + h^* > h^*$ 

But, consider two admissible estimates for length of ICAPS:

- $h_1$ :  $\geq$  2 days, because the workshops last 2 days
- $h_2$ :  $\geq$  3 days, because the main conference lasts 3 days
- $\rightsquigarrow$  We can infer that ICAPS lasts  $\geq$  5 days

Why? No "overlap" between the estimates.

Under certain conditions, the sum of two admissible heuristics is an admissible heuristic

We say that a set *H* of such heuristics is additive

June 2011

### Heuristics

### Additive Admissible Heuristics: Action Partitioning

Simple criterion for admissible additivity: Count cost of each action in at most one heuristic.

Given a task  $\Pi$  and a set of operators  $O_i \subseteq O$ , define the problem  $\Pi_{O_i}$  that is identical except

 $\mathit{cost}'(o) = \left\{ egin{array}{c} \mathit{cost}(o) & \mathrm{if} \ o \in O_i \ 0 & \mathrm{if} \ o 
ot \in O_i \end{array} 
ight.$ 

Action partitioning

An action partitioning is a set of problems  $\Pi_{O_1}, \ldots, \Pi_{O_n}$  resulting from a partitioning of the actions O of a planning task into disjoint sets  $O_1, \ldots, O_n$ .

Emil Keyder, Silvia Richter

Heuristics: 1. Introduction

Heuristics

### Additive Admissible Heuristics: Cost Partitioning

Action partitioning assigns the full cost of each operator to one problem, and zero to the others  $% \left( {{\left[ {{{\rm{cost}}} \right]}_{\rm{cost}}} \right)$ 

Instead, distribute the cost of each action among different problems  $\rightsquigarrow$  Ensure total across problems sums to at most cost of action

### Cost partitioning

A cost partitioning of  $\Pi$  is a set of problems  $\Pi_0, \ldots, \Pi_n$  that differ from  $\Pi$  only in operator costs, that satisfy

$$\sum_{i=0}^n \textit{cost}_i(o) \leq \textit{cost}(o)$$

where  $cost_i(o)$  is the cost of o in  $\Pi_i$ ,

Emil Keyder, Silvia Richter

June 2011

13 / 24

Heuristics: 1. Introduction

Additive Admissible Heuristics: Action Partitioning

Heuristics

### Additivity of action partitioning

Let  $\Pi_1, \ldots, \Pi_n$  be an action partitioning and let  $h_{\Pi_i}$  be an admissible estimate of the cost of  $\Pi_i$ . Then the sum of the heuristics  $h_{\Pi_i}$  is admissible:

 $\sum_{i=0}^n h_{\Pi_i}(s) \leq h^*(s)$ 

### Proof sketch

Optimal plan for  $\Pi$  is also plan for each  $\Pi_{O_i}$ , and the summed cost of these plans is  $h^*$ . Admissible estimates must therefore sum to less than  $h^*$ .

Emil Keyder, Silvia Richter

Heuristics: 1. Introduction

June 2011 14 / 24

Heuristics

### Additive Admissible Heuristics: Cost Partitioning

### Additivity of cost partitioning

Let  $\Pi_0, \ldots, \Pi_n$  be a cost partitioning of  $\Pi$  and let  $h_{\Pi_0}, \ldots, h_{\Pi_n}$  be admissible estimates for the costs of  $\Pi_0, \ldots, \Pi_n$ . Then the sum of the heuristics  $h_{\Pi_i}$  is admissible:

$$\sum_{i=0}^n h_{\Pi_i}(s) \leq h^*(s)$$

### Proof sketch

Optimal plan for  $\Pi$  is also plan for each  $\Pi_{O_i}$ , and the summed cost of these plans is  $h^*$ . Admissible estimates must therefore sum to less than  $h^*$ .

### Heuristics

### Heuristics: Consistency

### Definition

A heuristic h is consistent if  $s_g \in G \implies h(s_g) = 0$  and for all  $s \in S$  and edge (s, s'):

$$h(s) \leq cost(s,s') + h(s')$$

**Intuition:** A transition with cost c cannot decrease h by more than c

Emil Keyder, Silvia Richter

What kind of heuristic?

Efficiently computable

Planning as Heuristic Search

**Idea:** Search the state space using a heuristic

→ In practice, low order polynomial

Tradeoff between informativeness and computational effort

Evaluate few states at high computational cost, vs.

Hard to tell a priori whether computational effort will pay off

Evaluate many states at low computational cost

Heuristics: 1. Introduction

• Admissible for optimal solution, no guarantees required otherwise

Heuristics

June 2011 17 / 24

Heuristics Some properties Consistency implies admissibility  $ightarrow s, o \text{ s.t. } app(s, o) = s_g \in G, \ h(s) > h^*(s) \text{ would violate consistency}$ 

Induction on number of steps to goal

Admissible heuristics with  $\mathsf{A}^*$  and variants compute optimal solutions

▶ By the time we consider s<sub>g</sub> ∈ G, we have already considered all s that might be cheaper

Consistent heuristics with A\* guarantee optimal behaviour

Never expand same state twice

Emil Keyder, Silvia Richter

Heuristics: 1. Introduction

June 2011 18 / 24

### Problem Representation

### A Logistics Problem Example

Heuristics are obtained from factored representations of states:

- ► Set *V* of variables
- Domain  $D_v$  of values for each variable v



Heuristics: 1. Introduction



### **STRIPS**

Boolean variables:

•  $D_{truck-at-Freiburg} = \{true, false\}$ 

### Definition (STRIPS task)

- A STRIPS task  $\Pi = \langle F, s_0, G, O, cost \rangle$  is defined by: F A set of fluents/facts (boolean variables)  $s_0$  The initial state,  $s_0 \subseteq F$  G The goal description,  $G \subseteq F$  O The actions/operators, defined by tuples  $\langle pre(o), add(o), del(o) \rangle$ , each  $\subseteq F$  cost A function  $O \rightarrow \mathbb{R}_0^+$ • States are sets of fluents that have value *true* 
  - State space is potentially exponential:  $O(2^{|F|})$

### Emil Keyder, Silvia Richter

Heuristics: 1. Introduction

June 2011

21 / 24

Problem Representation

SAS<sup>+</sup> continued

- ▶ A full variable assignment assigns to each  $v \in V$  a value  $d \in D_v$  and represents a state
- $\blacktriangleright$  A partial variable assignment assigns values to a subset  ${\cal C} \subseteq {\cal V}$
- ▶ Sometimes refer to assignments (v = d) for  $v \in V$ ,  $d \in D_v$  as fluents/facts
- ▶ State space:  $\prod_{v \in V} |D_v|$



O The actions/operators, defined by tuples  $\langle pre(o), eff(o) \rangle$ , each a partial variable assignment

*cost* A function  $O \to \mathbb{R}_0^+$ 

Emil Keyder, Silvia Richter

Heuristics: 1. Introduction

June 2011 22 / 24

### Problem Representation

### Translation from STRIPS to SAS<sup>+</sup>

Easy to translate automatically from STRIPS to  $SAS^+$ 

**Idea:** Make a graph with node set F, and edges between any two fluents that cannot occur in the same state

► Example: ⟨*truck-at-Freiburg*, *truck-at-Brisbane*, *truck-at-Nancy*⟩

Cliques in graph represent finite-domain variables

Replace n boolean variables with a single n-valued variable

• More efficient representation – n vs.  $2^n$  configurations



### Π<sup>+</sup> Definition

### The Delete Relaxation $\Pi^+$ – Formalization

In STRIPS, remove delete lists:

### Definition

Given a STRIPS problem  $\Pi = \langle F, s_0, G, O, cost \rangle$ , its delete relaxation is given by  $\Pi^+ = \langle F, s_0, G, O', cost \rangle$ , where

$$O' = \{ \langle Pre(o), Add(o), \emptyset \rangle \mid o \in O \}$$

- $\triangleright$  |s| increases with each action
- Stratification of fluents by first level at which they can be achieved



June 2011

5 / 49

Emil Keyder, Silvia Richter

Heuristics: 2. Delete Relaxation

 $P^{+} P^{+} Difficulty$ Simplifying Π<sup>+</sup>  $Ideally, |s_{0}| = |Pre(o)| = |Add(o)| = |G| = 1$ • Shortest path problem on graph with V = F, E = O $\Rightarrow$  polynomial in F and E• |F| is small, so easily solvable
Question: Does there exist a Π<sup>+'</sup> equivalent to Π<sup>+</sup> with this property?
Answer: Almost (except for |Pre(o)|)
Theorem
For any Π<sup>+</sup>, there exists an equivalent problem Π<sup>+'</sup> such that  $|s'_{0}| = 1$ , |G'| = 1, and |Add(o)| = 1 for all  $o \in O'$ 









Approximating  $h^+$ Several heuristics defined in terms of method used to estimate the costs of these sets To be optimal, must take into account interactions between plans for fluents in set  $\rightsquigarrow$  Intractable in general Instead, make the independence assumption: Independence Assumption The (cost of) the best plan for a set of fluents *P* can be estimated as a function of the (costs of) the best plans for each fluent  $p \in P$ Suboptimal, but makes the problem tractable



### Polynomial Approximation Approaches

Theorem For any set of fluents  $P \subseteq F$ ,

$$\max_{p\in P} h^+(p) \le h^+(P) \le \sum_{p\in P} h^+(p)$$

- 1. Prove a minimum cost for any solution
  - Lower bound, admissible heuristic for optimal planning
- 2. Find a solution to  $\Pi^+$  with no guarantees of optimality
  - Upper bound, inadmissible heuristic for satisficing planning

Emil Keyder, Silvia Richter

Heuristics: 2. Delete Relaxation

h<sup>max</sup> & h<sup>add</sup> h<sup>max</sup>

The Max Heuristic  $h^{\max}$ 

$$h^{\max}(s) = h^{\max}(G; s)$$

$$h^{\max}(P;s) = \max_{p \in P} h^{\max}(p;s)$$

$$h^{\max}(p; s) = \begin{cases} 0 & \text{if } p \in s \\ \min_{\{o \mid p \in add(o)\}} [cost(o) + h^{\max}(pre(o); s)] & \text{otherwise} \end{cases}$$

h<sup>max</sup> & h<sup>add</sup> h<sup>max</sup>

### The Max Heuristic $h^{\max}$

An admissible heuristic that gives a lower bound on  $h^+$  (Bonet & Geffner, 2001)

**Intuition:** Estimates cost of a set as the cost of the most expensive fluent in set:

$$h^{\max}(P;s) = \max_{p \in P} h^{\max}(p;s)$$

Drawbacks:

Loose lower bound, uninformative

 $h^{\max}$  is an instance of a family of heuristics – more on this later

Emil Keyder, Silvia Richter

Heuristics: 2. Delete Relaxation

June 2011 14 / 49

### h<sup>max</sup> & h<sup>add</sup> h<sup>add</sup>

### The Additive Heuristic $h^{\text{add}}$

First practical domain-independent planning heuristic for satisficing planning (Bonet & Geffner, 2001)

Intuition: Estimates cost of a set as the sum of the costs of the elements:

$$h^{\mathrm{add}}(G) = \sum_{g \in G} h^{\mathrm{add}}(g)$$

Assumes **no** positive interactions

- Inadmissible, so not useful for optimal planning
- Better than h<sup>max</sup> for satisficing planning

Emil Keyder, Silvia Richter

Heuristics: 2. Delete Relaxation

June 2011



h<sup>max</sup> & h<sup>add</sup> Computation Computation function h(G; s) begin for  $p \in F$  do /\* initialization \*/ if  $p \in s$  then h(p) = 0 else  $h(p) = \infty$ end repeat a = chooseAction() for  $q \in Add(a)$  do if h(a; s) < h(q; s) then h(q; s) = h(a; s)endif end until fixpoint return h(G; s)end

























Relaxed Plan Heuristics

Relaxed Plan Heuristics Relaxed Planning Graph ► Tool to graphically represent heuristic computation ▶ Layer *i* contains facts achievable with *i* layer parallel plan g С b<sub>o</sub> c<sub>o</sub> c<sub>o</sub> d<sub>o</sub> d<sub>o</sub> e 。 e<sub>o</sub>  $G_{\circ}$ Go Go Heuristics: 2. Delete Relaxation 31 / 49

hF

Relaxed Plan Heuristics The FF Heuristic

Construct relaxed plan by starting from goal, choose supporter for each fluent at first layer in which it appears







Relaxed Plan Heuristics Relaxed Plan Heuristics - Benefits

Generate an explicit plan  $\pi$  whose cost is the heuristic value, rather than just a numeric estimate

Advantages:

- Explicit representation of plan no overcounting!
- Helpful actions: Operators likely to lead to better state  $\rightsquigarrow$  Used by search algorithms to generate/evaluate fewer nodes

Emil Keyder, Silvia Richter

Heuristics: 2. Delete Relaxation

June 2011 34 / 49

Relaxed Plan Heuristics  $\pi(h^{\text{add}}) \& \pi(h^{\text{max}})$ 

### Relaxed Plans from $h^{\text{max}}$ , $h^{\text{add}}$ , etc.

 $h^{\mathsf{FF}}$  uses uniform cost  $h^{\mathsf{max}}$  supporters + plan extraction algorithm

**Drawback:** Uniform cost  $h^{\text{max}}$  not cost-sensitive

Solution: Cost-sensitive heuristic to choose best supporter (Keyder & Geffner, 2008, others) ...

$$o_p(s) = \operatorname*{argmin}_{\{o \mid p \in add(o)\}} cost(o) + h(pre(o); s)$$

- ... combined with generic plan extraction algorithm
  - Construct  $\pi^+$  by collecting the best supporters recursively backwards from the goal ...

Emil Keyder, Silvia Richter



### Relaxed Plan Extraction Algorithm







### The Set-Additive Heuristic $h_{set}^{add}$ $h_{set}^{add}(s) = Cost(\pi(G; s))$ $\pi(P; s) = \bigcup_{p \in P} \pi(p; s)$ where $\pi(p; s) = \begin{cases} \{\} & \text{if } p \in s \\ \pi(o_p(s); s) & \text{otherwise} \end{cases}$ $Cost(\pi) = \sum_{a \in \pi} cost(a)$ $o_p(s) = \underset{\{o | p \in add(o)\}}{\operatorname{argmin}} \left[ Cost(\{o\} \cup \bigcup_{q \in pre(o)} \pi(q; s)) \right]$

Heuristics: 2. Delete Relaxation

40 / 49

June 2011

Emil Keyder, Silvia Richter

### Relaxed Plan Heuristics h<sub>set</sub><sup>add</sup>

### The Set-Additive Heuristic $h_{\text{set}}^{\text{add}}$

**Intuition:** Estimate the cost of sets considering overlap between plans for individual fluents

Potentially better estimates

**Drawback:**  $\cup$  operation expensive compared to cheaper  $\sum$  or max operation

Emil Keyder, Silvia Richter

Heuristics: 2. Delete Relaxation

June 2011 41 / 49











### The Steiner Tree Problem

### The Steiner Tree Problem

Given a graph  $G = \langle V, E \rangle$  and a set of *terminal nodes*  $T \subseteq V$ , find a minimum-cost tree S that spans all  $t \in T$ 

When T = V, this is the tractable Minimum Spanning Tree (MST) problem

Otherwise, equivalent to finding best set of non-terminal nodes  ${\it P}$  to span, known as Steiner Points

• Steiner Tree is then MST over  $P \cup T$ 

Relaxed Plan Heuristics Independence Remaining issues Relaxed-plan heuristics solve the overcounting issue by computing an explicit relaxed plan with no duplicate actions Independence assumption issues remain



Relaxed Plan Heuristics Independence

### Improving Steiner Trees

### The Steiner Tree heuristic

Exploits parallels between Steiner trees and relaxed plans, and MST property of Steiner trees, to obtain an improvement algorithm for relaxed plans (Keyder & Geffner, 2009)

Start with a suboptimal relaxed plan, and attempt to get a better one

General idea:

- 1. Remove a partial plan  $\pi'$  from the relaxed plan
- 2. Try to find a new partial plan  $\pi^{\prime\prime}$  with lower cost that makes it whole again

Intuition: Make relaxed plan closer to MST

### Conclusions Conclusions • The optimal delete relaxation heuristic $h^+$ is admissible and informative, but calculating it is NP-hard Sub-optimal solutions have proven to be effective heuristics for satisficing planning • $h^{\text{add}}$ suffers from two problems: 1. Overcounting of actions when combining estimates 2. The independence assumption • Overcounting problem can be solved with relaxed-plan heuristics → Cost-sensitive best supporters • Steiner Tree heuristic is an attempt at improving estimates obtained with the independence assumption Heuristics: 2. Delete Relaxation

June 2011

49 / 49

Emil Keyder, Silvia Richter



 $h^{\max} = h^1$ 

 $h^{\max}$  estimates cost of set *P* as max cost of any  $p \in P$ :

$$h^{\max}(P;s) = \max_{p \in P} h^{\max}(p;s)$$

 $h^{\max} = h^1$ 

Alternatively:

$$h^1(P;s) = \max_{P' \subseteq P \land |P'| \leq 1} h^1(P';s)$$

Idea: Generalize to arbitrary m

Emil Keyder, Silvia Richter

Heuristics: 3. Critical Path

$$h^{max} = h^1$$
  
 $h^m$  Heuristics  
The  $\Pi^m$  Compilation  
Summary  
Emil Keyder, Silvia Richter Heuristics: 3. Critical Path June 201 2 / 13

### h<sup>m</sup> Heuristics

Consider subsets of *P* of size *m*:

$$h^m(P;s) = \max_{P' \subseteq P \land |P'| \le m} h^m(P';s)$$

Estimate cost of P as most expensive subset of size m or less (Haslum & Geffner, 2000)

**Question:** How can a set of fluents P' of size m be made true?

hm Heuristics

Two ways:

$$L \{a \mid P' \subseteq add(a)\}$$

2.  $\{a \mid P' \cap add(a) \neq \emptyset \land P' \cap del(a) = \emptyset\}$ 

 $h^m$  heuristics take into account delete information



 $h^m(s) = h^m(G;s)$ 

$$h^{m}(P;s) = \begin{cases} 0 & \text{if } P \subseteq s \\ \min_{a}(cost(a) + h^{m}(R(P,a);s)) & \text{if } |P| \leq m) \\ \max_{P' \subseteq P \land |P'| \leq m} h^{m}(P';s) & \text{otherwise} \end{cases}$$

Computation is polynomial for fixed m, exponential in m in general

- Number of subsets of size  $\leq m$  is  $O(|F|^m)$
- Polynomial can still be very expensive

Too slow to compute in every state

- ► Used in backwards search
- And to compute mutexes

### h<sup>m</sup> Heuristics

### Mutexes

The  $h^2$  heuristic is commonly used to compute mutexes

- $h^2(\{p,q\};s_0) = \infty \implies p \text{ and } q \text{ are mutex}$
- Mutexes give finite-domain variables

Mutex detection is sound but not complete (for fixed m)

- Assume  $h^3(\{p, q, r\}) = \infty$ , but  $h^2(\{p, q\})$ ,  $h^2(\{p, r\})$ ,  $h^2(\{q, r\})$  are all finite.
- If the only action making {s, t} true has precondition {p, q, r}, h<sup>2</sup>({s, t}) is finite, but s and t are mutex.

Emil Keyder, Silvia Richter

Heuristics: 3. Critical Path

June 2011 9 / 13

The  $\Pi^m$  Compilation

### The $\Pi^m$ Compilation

The  $\Pi^m$  task is a planning task that encodes  $h^m$  information (Haslum, 2009)

- Fluents of  $\Pi^m$  are sets of fluents in  $\Pi$
- One operator in Π → many operators in Π<sup>m</sup>, each with different context
  - $\rightsquigarrow$  Fluents that were true in addition to pre(a) when a was applied
- Consider deletes when constructing  $\Pi^m$  actions

### Key properties:

- $\blacktriangleright h^1(\Pi^m) = h^m(\Pi)$
- No deletes in  $\Pi^m$

Emil Keyder, Silvia Richter

Heuristics: 3. Critical Path

Properties For large enough m,  $h^m = h^*$ • In the worst case need m = |F|  $h^+$  and  $h^m$  are both admissible heuristics, but incomparable •  $h^m$  is not a delete relaxation heuristic Market end end to be the end of the

### The Π<sup>m</sup> Compilation

### Formal Definition

### Definition $(\Pi^m)$

Given a STRIPS task  $\Pi = \langle F, s_0, G, O, cost \rangle$ , its  $\Pi^m$  compilation is given by a STRIPS task  $\Pi^m = \langle F^m, s_0^m, G^m, O^m, cost \rangle$ , where  $F^m = \{\pi_C \mid C \subseteq F \land |C| \leq m\}, s_0^m$  and  $G^m$  are defined analogously, and

$$O^m = \{o_C \mid o \in O \land C \in contexts(o)\}$$

where

$$contexts(o) = \{C \in F^{m-1} \mid C \cap add(o) = C \cap del(o) = \emptyset$$
  

$$pre(o_C) = \{\pi_C \mid C \subseteq (pre(o) \cup C) \land |C| \le m\}$$
  

$$add(o_C) = \{\pi_C \mid C \subseteq (add(o) \cup C) \land |C| \le m\}$$
  

$$del(o_C) = \emptyset$$

~					
ווכ	m	m	а	rv	
			_	• •	

### Conclusions

- h<sup>m</sup> heuristics are admissible estimates obtained by considering critical paths
- $h^m$  heuristics take deletes into account, and are incomparable to  $h^+$
- Their computation is polynomial for fixed *m*, but exponential in *m* in general
- As  $m \to |F|$ ,  $h^m(s) \to h^*(s)$

Emil Keyder, Silvia Richter

Heuristics: 3. Critical Path

June 2011 13 / 13



Background

Background

 $h^{\text{cea}}$  (Helmert & Geffner, 2008) is a reformulation of an earlier heuristic,  $h^{\text{CG}}$  (Helmert, 2004)

- Replaces algorithm with recursive equations, clarifying connection to h<sup>add</sup>
- Removes limitations on problem structure

 $h^{\text{cea}}$  is defined in terms of finite-domain variables (SAS<sup>+</sup>)

**Intuition:** To achieve values for a set of variables, achieve one of them first

 $\rightsquigarrow$  Compute cost of the other variables considering side effects of achieving the first one



### $h^{\text{add}}$ in SAS<sup>+</sup>

 $h^{\text{add}}$  can be written for SAS<sup>+</sup> problems as follows:

Background

$$h^{\text{add}}(G; s) = \sum_{x \in G} h^{\text{add}}(x; x_s)$$
$$h^{\text{add}}(x \mid x_s) = \begin{cases} 0 & \text{if } x = x_s \\ \\ \min_{o: z \to x} \left[ cost(o) + \sum_{x_i \in z} h^{\text{add}}(x_i \mid x_{is}) \right] & \text{if } x \neq x_s \end{cases}$$

where  $o: z \to x$  is an action that assigns x and has z as a precondition, and  $x_s$  is the value of the variable of x in s.

Heuristics: X. h<sup>cea</sup>

### Background

### $h^{\text{add}}$ in SAS<sup>+</sup>

 $h^{\text{add}}$  can be written for SAS<sup>+</sup> problems as follows:

where  $o: z \to x$  is an action that assigns x and has z as a precondition, and  $x_s$  is the value of the variable of x in s.

**Idea of**  $h^{\text{cea}}$ : Take into account side effects by evaluating precondition  $x_i$  in a state s' that is different from s

```
Emil Keyder, Silvia Richter
```

Heuristics: X. h<sup>cea</sup>

$$h^{\text{cea}} = \frac{h^{\text{cea}} \text{ Equations}}{h^{\text{cea}}(x \mid x')} = \begin{cases} 0 & \text{if } x = x' \\ \min_{o:x'',z \to x} \left[ cost(o) + h^{\text{cea}}(x'' \mid x') \\ + \sum_{x_i \in z} h^{\text{cea}}(x_i \mid x'_i) \right] & \text{if } x \neq x' \end{cases}$$

The cost of achieving the preconditions is expressed as the heuristic cost of achieving  $x'' \dots$ 

$$x' \longrightarrow \ldots \longrightarrow x'' \stackrel{o}{\longrightarrow} x$$

$$h^{\text{cea}} \text{ Equations}$$

$$h^{\text{cea}} = \begin{cases} 0 & \text{if } x = x' \\ n^{\text{cea}}(x \mid x') = \begin{cases} 0 & \text{if } x = x' \\ o:x'',z \to x & [cost(o) + h^{\text{cea}}(x'' \mid x') \\ + \sum_{x_i \in z} h^{\text{cea}}(x_i \mid x_i') & \text{if } x \neq x' \end{cases}$$

$$h^{\text{cea}} \text{ considers } pre(o) \text{ in two parts:}$$

$$h^{\text{cea}} \text{ considers } pre(o) \text{ in two parts:}$$

$$h^{\text{cea}} \text{ considers } x'', \text{ defined on the same variable as } x$$

$$z, \text{ the rest of the precondition}$$

$$h^{\text{cea}} \text{ Equations}$$

$$h^{\text{cea}} \left( x \mid x' \right) = \begin{cases} 0 & \text{if } x = x' \\ \min_{o:x'',z \to x} \left[ cost(o) + h^{\text{cea}}(x'' \mid x') \\ + \sum_{x_i \in z} h^{\text{cea}}(x_i \mid x'_i) \right] & \text{if } x \neq x' \end{cases}$$

Heuristics: X. hcea

... plus the heuristic cost of achieving the other preconditions from their values  $x'_i$  that result from achieving x''

The values  $x'_i$  are the values in the projected state s(x'' | x')

**Assumption:** x'' is achieved first – The pivot condition

$$x' \longrightarrow \ldots \longrightarrow x'' \stackrel{o}{\longrightarrow} x$$

Emil Keyder, Silvia Richter

Heuristics: X. hcea

June 2011

5 / 17

Emil Keyder, Silvia Richter

Heuristics: X. h<sup>cea</sup>

June 2011

### h<sup>cea</sup> Equations

### How to Compute $s(x'' \mid x')$

Equations defining  $h^{cea}$  and s(x'' | x') are mutually recursive

Let  $o: x''', z' \to x'', y_1, \dots, y_n$  be the operator that results in minimum  $h^{cea}$  value for x''

 $\rightsquigarrow$  Similar to best supporters in  $\Pi^+$ 

**Notation:** s[x'] denotes s "overridden" with x'

 $s(x'' \mid x') = \begin{cases} s[x'] & \text{if } x'' = x' \\ s(x''' \mid x')[z'][x'', y_1, \dots, y_n] & \text{if } x'' \neq x' \end{cases}$   $x' \dots \longrightarrow x''' \xrightarrow{o} x'' \longrightarrow x \\ s \dots \longrightarrow s(x''' \mid x') \xrightarrow{o} s(x'' \mid x') \longrightarrow s(x \mid x')$ 

Emil Keyder, Silvia Richter

Heuristics: X. h<sup>cea</sup>

h<sup>cea</sup> Equations

h<sup>cea</sup>

The value of  $h^{cea}$  for the set of goals G is the same as  $h^{add}$ :

$$h^{\mathsf{cea}}(s) = \sum_{x \in G} h^{\mathsf{cea}}(x \mid x_s)$$

Sum costs of achieving goal value for each variable in the goal given its value  $x_s$  in s

# $How to Compute s(x'' \mid x')$ The state $s(x'' \mid x')$ is defined recursively beginning from $s(x''' \mid x')...$ ... and updated with $pre(o) \setminus \{x'''\} = z' ...$ $\Rightarrow$ known to be true, since precondition of o... and finally with $eff(o) = x'' \cup \{y_1, ..., y_n\}$ $\Rightarrow$ known to be true, since effect of o $s(x'' \mid x') = \begin{cases} s[x'] & \text{if } x'' = x' \\ s(x''' \mid x')[z'][x'', y_1, ..., y_n] & \text{if } x'' \neq x' \end{cases}$ $x' ... \Rightarrow x''' & a x x \\ s ... & s(x''' \mid x') & a s(x''' \mid x') & a s(x' \mid x') \end{cases}$ Emil Keyder, Silvia Richter Heuristics X. here

### $\mu^{cea} \text{ Equations}$ Example: Computation of $h^{cea}$ Let $\Pi = \langle V, O, s_0, G, cost \rangle$ be an SAS<sup>+</sup>problem with $V X = \{x_0, \dots, x_n\}$ $Y = \{true, false\}$ $O a : \{\neg y\} \rightarrow \{y\} b_i : \{y, x_i\} \rightarrow \{\neg y, x_{i+1}\}$ $s_0 \{x_0, y\}$ $G \{x_n\}$ $cost \ c(a) = c(b_i) = 1$ The optimal plan is then $\pi^* = \langle b_0, a, \dots, a, b_{n-1} \rangle$

containing  $n \times b_i + (n-1) \times a = 2n-1$  actions

Emil Keyder, Silvia Richter

June 2011 9 / 17



h<sup>cea</sup> Equations

Example: Computation of  $h^{cea}$ 

We have:

$$h^{\text{cea}}(x_1 \mid x_0) = 1$$
  
 $h^{\text{cea}}(x_n \mid x_0) = h^{\text{cea}}(x_{n-1} \mid x_0) + 2$ 

 $h^{\text{cea}}$  gives optimal solution to this problem:

$$h^{\text{cea}}(x_n \mid x_0) = 2(n-1) + 1 = 2n-1$$
  
 $h^{\text{cea}}(s) = h^*(s)$ 

h<sup>cea</sup> Equations

### Example: Computation of $h^{cea}$

What is the value of  $h^{cea}(x_i \mid x_0)$ ?

$$\begin{aligned} h^{\text{cea}}(x_{0} \mid x_{0}) &= & 0 \\ h^{\text{cea}}(x_{1} \mid x_{0}) &= & c(b_{0}) + h^{\text{cea}}(x_{0} \mid x_{0}) + h^{\text{cea}}(y \mid y_{s(x_{0} \mid x_{0})}) \\ &= & 1 + 0 + 0 \\ h^{\text{cea}}(x_{i} \mid x_{0}) &= & c(b_{i-1}) + h^{\text{cea}}(x_{0} \mid x_{i-1}) + h^{\text{cea}}(y \mid y_{s(x_{i-1} \mid x_{0})}) \\ &= & c(b_{i-1}) + h^{\text{cea}}(x_{0} \mid x_{i-1}) + h^{\text{cea}}(y \mid y_{\{x_{0}, \neg y\}}) \\ &= & c(b_{i-1}) + h^{\text{cea}}(x_{0} \mid x_{i-1}) + h^{\text{cea}}(y \mid \neg y) \\ &= & 1 + h^{\text{cea}}(x_{0} \mid x_{i-1}) + 1 \end{aligned}$$
Since  $s(x_{i-1} \mid x_{0}) = \{x_{i-1}, \neg y\}$ , y evaluated from value  $\neg y$ 
  
Emil Keyder, Silvia Richter Heuristics: X.  $h^{\text{cea}}$  Jue 201 14 / 17

### Conclusions

### Observations

Depends on finite-domain variables

• With boolean variables, equivalent to  $h^{\text{add}}$ 

$$x' \longrightarrow \ldots \longrightarrow x''$$
(false)  $\longrightarrow x$ (true)

Must be the case that x' = x'' = false, so s(x''|x') = s

Preserves information about changes in a single variable from s

$$\ldots + \sum_{x_i \in z} h^{\text{cea}}(x_i \mid x'_i)$$

Fixed order in which preconditions are assumed to be achieved  $\rightsquigarrow$  Always achieve value of same variable first

Emil Keyder, Silvia Richter

Heuristics: X. h<sup>cea</sup>

	Conclusions		
Conclusions			
Can be more informative that	an $h^+$		
$\rightsquigarrow$ But not in general compa	arable		
Possible improvements:			
<ul> <li>Consider precedences ir Helmert, 2009)</li> </ul>	the preconditions (Cai,	Hoffmann, &	
<ul> <li>Consider projected state variables rather than a</li> </ul>	es that differ from <i>s</i> in a single one	bounded number	r of
Emil Keyder, Silvia Richter	Heuristics: X. h <sup>cea</sup>	June 2011	17 / 17



What Landmarks Are Landmarks

Landmarks

A landmark is a property of every plan for a planning task

- 1. Propositional landmark: a formula  $\phi$  over the set of fluents
  - $\rightsquigarrow \phi$  is made true in some state during the execution of any plan
- 2. Action landmark: a formula  $\psi$  over the set of actions
  - $\rightsquigarrow \psi$  is made true by any plan interpreted as a truth assignment to its set of actions

Landmarks can be (partially) ordered

Some landmarks and orderings can be discovered automatically

 $\rightsquigarrow$  Mostly restricted to single fact/action landmarks or simple formulas (disjunctions/conjunctions)

Landmarks are used in some state-of-the-art heuristics

Emil K	ovdor	Silvia	Richter

### What Landmarks Are

Landmarks Landmark Orderings Complexity

### Landmark Discovery

Theory Backchaining Forward Propagation & Declarative Characterization  $\Pi^m$  Landmarks Orderings

### Landmark-Counting Heuristics

The LAMA Heuristic  $h^{LM}$ The Admissible Landmark Heuristic  $h^{LA}$ 

### The Landmark Cut Heuristic

ldea Example

### Emil Keyder, Silvia Richter

Heuristics: 5. Landmark Heuristics

June 2011 2 / 52



### What Landmarks Are Landmarks

### Landmarks from Other Landmarks

Propositional landmarks imply action landmarks

► A single fact landmark *p* implies the disjunctive action landmark

 $\{a|p\in add(a)\}$ 

Action landmarks imply propositional landmarks

A single action landmark a implies the propositional landmarks



Emil Keyder, Silvia Richter

Heuristics: 5. Landmark Heuristics

5 / 52

June 2011





► Reasonable ordering A →<sub>r</sub> B, iff given B was achieved before A, any plan must delete B on the way to A, and re-achieve B

$$B \rightsquigarrow \neg B \rightsquigarrow A \rightsquigarrow B \implies A \rightarrow_r B$$

- Initial state landmarks can be reasonably ordered after other landmarks (e.g., if they must be made false and true again)
- This can never happen with sound orderings

what Landmarks Are Complexit	What Landmarks Are	Complexity
------------------------------	--------------------	------------

### Landmark Complexity

### **Basic Result**: Everything is **PSPACE-complete**

- Deciding if a propositional formula is a landmark is PSPACE-complete ~> Proof Sketch: Same problem as deciding if the problem without actions that achieve this fact is unsolvable
- Deciding if there is a natural / necessary / greedy-necessary / reasonable ordering between two landmarks is PSPACE-complete

Emil Keyder, Silvia Richter

Heuristics: 5. Landmark Heuristics



### Landmark Discovery in Theory

### Theory

A is a fact landmark  $\iff \pi_A$  is unsolvable

where  $\pi_A$  is a  $\pi$  with all actions that make A true removed

The delete relaxation  $\pi_A^+$  is unsolvable  $\implies \pi_A$  is unsolvable

- > This can be used to obtain delete-relaxation landmarks
- But more efficient methods exist

### Emil Keyder, Silvia Richter

Heuristics: 5. Landmark Heuristics

June 2011 10 / 52



June 2011



Landmark Discovery Forward Propagation & Declarative Characterization

### Landmark Discovery II: Forward Propagation

Find landmarks by forward propagation in relaxed planning graph (Zhu & Givan 2003)

- Finds all causal delete-relaxation landmarks in polynomial time (where causal means: appearing in preconditions for actions)
- Propagate information on necessary predecessors
  - Label each fact node with itself
  - Propagate labels along arcs

 Landmark Discovery
 Backchaining

 Landmark Discovery I: Backchaining (ctd.)

 Pro:

 • Finds disjunctive landmarks as well as facts

 Con:

 • No criterion for which backchaining is complete for П<sup>+</sup>

 • Requires arbitrary limits e. g. on size/form of disjunctions

### Emil Keyder, Silvia Richter

Heuristics: 5. Landmark Heuristics

June 2011 14 / 52

Landmark Discovery Forward Propagation & Declarative Characterization

### Landmark Discovery II (ctd.)

Actions (numbers): propagate union over labels on preconditions — all preconditions are necessary

Facts (letters): propagate intersection over labels on achievers

- only what's necessary for all achievers is necessary for a fact



No-ops and repeated nodes not shown



18 / 52

June 2011

### Landmark Discovery II: Complete Equations

The Zhu & Givan method can be seen as computing the fixpoint solution to a set of equations (Keyder et al., 2010):





### Landmark Discovery Orderings

### **Finding Orderings**

Natural and (greedy-)necessary orderings are found along with landmarks

- A is a landmark for  $B: A \rightarrow B$
- ► *A* is a precondition for all achievers *a* of *B* that do not have *B* as a landmark:  $A \rightarrow_{gn} B$

If reasonable orderings used, they are computed in a post-processing step.

Heuristics: 5. Landmark Heuristics

- Not discussed here
- See landmark tutorial from ICAPS 2010

Landmark-Counting Heuristics

Using Landmarks

Emil Keyder, Silvia Richter

- Some landmarks and orderings can be discovered efficiently
- So what can we do once we have these landmarks?
- Previous use as subgoals for search control (Hoffmann et al., 2004)
- Here, we focus on use for deriving heuristic estimates
- Next section assumes landmarks are computed once for initial state
- Recomputing for every state would be more informative but costly

### Conter Methods for Landmark Finding Find landmarks via domain transition graphs (Richter et al. 2008) Not discussed here See landmark tutorial from ICAPS 2010

Heuristics: 5. Landmark Heuristics

## The LAMA Heuristic *h*<sup>LM</sup> Using Landmarks for Heuristic Estimates The number of landmarks that still need to be achieved is a heuristic estimate (Richter and Westphal 2010) Used by LAMA - winner of the IPC-2008 sequential satisficing track Inadmissible heuristic, because an action may achieve more than one landmark

21 / 52

June 2011

Emil Keyder, Silvia Richter

June 2011





- Heuristic value of a goal state may be non-zero (if the plan found does not obey all reasonable orderings, and consequently not all landmarks are accepted).
   Solution: explicitly test states for goal condition
- The definition of reasonable orderings allows an ordering  $A \rightarrow_r B$  if A and B become true simultaneously. Two solutions:
  - Accept a landmark if it has been made true at the same time as its predecessor (Buffet and Hoffmann, 2010)
  - Modify the definition of reasonable orderings to disallow such orderings





Proof: A is a landmark, therefore it needs to be true in all plans, including plans that start with π<sub>2</sub>

### Landmark-Counting Heuristics The LAMA Heuristic hLM

### Fusing Data from Multiple Paths

- Improvement to path-dependent landmark heuristics (Karpas and Domshlak, 2009)
- Suppose  $\mathscr{P}$  is a set of paths from  $s_0$  to a state s. Define

$$L(s,\mathscr{P}) = (L \setminus \mathsf{Accepted}(s,\mathscr{P})) \cup \mathsf{ReqAgain}(s,\mathscr{P})$$

### where

- Accepted $(s, \mathscr{P}) = \bigcap_{\pi \in \mathscr{P}} \operatorname{Accepted}(s, \pi)$
- ► ReqAgain(s, 𝒫) ⊆ Accepted(s, 𝒫) is specified as before by s and the various rules
- L(s, P) is the set of landmarks that we know still needs to be achieved after reaching state s via the paths in P

Emil	Kevder	Silvia	Richter
L	Reyuer,	Olivia	rucitei

Heuristics: 5. Landmark Heuristics

June 2011 33 / 52

Landmark-Counting Heuristics The Admissible Landmark Heuristic hLA

### **Ensuring Admissibility**

> Actions share their cost between all the landmarks they achieve

$$\forall o \in O: \sum_{B \in L(o|s,\mathscr{P})} \mathit{cost}(o,B) \leq \mathit{cost}(o)$$

cost(o, B): cost "assigned" by action *o* to *B*  $L(o|s, \mathscr{P})$ : the set of landmarks achieved by *o* 

> The cost of a landmark is the minimum cost assigned to it by any action

Then

$$h^{\mathsf{L}}(s,\pi) := cost(L(s,\pi)) = \sum_{B \in L(s,\pi)} cost(B)$$

is admissible

Emil Keyder, Silvia Richter

Heuristics: 5. Landmark Heuristics

Landmark-Counting Heuristics The Admissible Landmark Heuristic h Admissible Heuristic Estimates • LAMA's heuristic  $h^{LM}$ : the number of landmarks that still need to be achieved ► *h*<sup>LM</sup> is inadmissible - a single action can achieve multiple landmarks Example: hand-empty and on-A-B can both be achieved by stack-A-B Admissible heuristic: assign a cost to each landmark, sum over the costs of landmarks (Karpas and Domshlak, 2009) Emil Keyder, Silvia Richter Heuristics: 5. Landmark Heuristics 34 / 52 June 2011 Landmark-Counting Heuristics The Admissible Landmark Heuristic h Cost Sharing - How? How to share action costs between landmarks? Easy answer: uniform cost sharing - each action shares its cost equally between the landmarks it achieves  $cost(o, B) = \frac{cost(o)}{|L(o|s, \pi)|}$ 

### Landmark-Counting Heuristics The Admissible Landmark Heuristic h

### **Uniform Cost Sharing**

- Advantage: Easy and fast to compute
- Disadvantage: can be much worse than the optimal cost partitioning
- Example: all actions cost 1 uniform cost sharing





### $h^{\rm L} = 4$ uniform $h^{\rm L} = 2.5$ min(1)=1 $a_4$ $p_4$ $p_3$ min(1)=1 $a_3$ $p_2$ min(1)=1 $a_2$ min(1)=1 $a_1$ $\min(0,0,0,0)=0$ Emil Keyder, Silvia Richter Heuristics: 5. Landmark Heuristics June 2011 The Admissible Landmark Heuristic hLA Landmark-Counting Heuristics How can we do better? So far: Uniform cost sharing is easy to compute, but suboptimal Optimal cost sharing takes a long time to compute Q: How can we get better heuristic estimates that don't take a long time to compute? A: Exploit additional information - action landmarks Emil Keyder, Silvia Richter Heuristics: 5. Landmark Heuristics June 2011

38 / 52

40 / 52

### Landmark-Counting Heuristics The Admissible Landmark Heuristic hLA

### **Uniform Cost Sharing**

- Advantage: Easy and fast to compute
- Disadvantage: can be much worse than the optimal cost partitioning
- Example: all actions cost 1 optimal cost sharing





Land	mark-Counting Heuristics	The Admissible Landmark Heuristic h <sup>LA</sup>		
Summary				
Carrinary				
Landmarks describ	e the implicit st	ructure of a planning tas	k	
<ul> <li>Can be used to der heuristics</li> </ul>	rive admissible a	and inadmissible landma	ırk-counting	J
These heuristics ar	e path-depende	ent		
Emil Keyder, Silvia Richter	Heuristics: 5. Lan	dmark Heuristics	June 2011	42 / 52
Th	e Landmark Cut Heuristic	ldea		
The Level of O				
The Landmark Cut	Heuristic (c	cont.)		

Idea (Helmert & Domshlak, 2009):

- Use critical paths (from h<sup>max</sup> computation) to find disjunctive action landmarks
- Extract landmarks iteratively, subtracting their cost from h<sup>max</sup> estimates
- ► Cost of each landmark := cost of cheapest action in landmark
- Heuristic value := sum over all landmark costs

### The Landmark Cut Heuristic Idea

### The Landmark Cut Heuristic (cont.)

Computation in a nutshell:

- 1. Compute  $h^{max}$  costs for all facts
- 2. Reduce goals and preconditions to singleton sets in a way that preserves  $h^{\text{max}}$
- Replace all multi-effect operators by a set of unary operators in a way that preserves h<sup>max</sup>
- 4. Compute the justification graph for the resulting task, where  $h^{\text{max}}$  values are shortest distances
- 5. Compute a cut in the justification graph that separates before-goal zone from goal zone.
- 6. The cut corresponds to one disjunctive action landmark in the result Extract it through cost partitioning
- 7. Start from the beginning until  $h^{\text{max}}$  values are 0

Emil Keyder, Silvia Richter

Heuristics: 5. Landmark Heuristics

June 2011 45 / 52

The Landmark Cut Heuristic Example

### *h*<sup>LM-cut</sup> Example (cont.)

- 1. Compute  $h^{max}$  costs for all facts
- 2. Reduce goals and preconditions to singleton sets in a way that preserves h<sup>max</sup>
- Replace all multi-effect operators by a set of unary operators in a way that preserves h<sup>max</sup>



### The Landmark Cut Heuristic Example

### *h*<sup>LM-cut</sup> Example

Transport object from A to B, by truck or by teleportation (Partial) RPG:



### The Landmark Cut Heuristic Example

### *h*<sup>LM-cut</sup> Example (cont.)

- 4. Compute the justification graph for the resulting task, where  $h^{max}$  values are shortest distances
- 5. Compute a cut in the justification graph that separates before-goal zone from goal zone.









Example

### Conclusion

Clever, heuristically guided way of discovering landmarks

The Landmark Cut Heuristic

- $h^{\text{LM-cut}}$  approximates  $h^+$  very closely
- Landmarks are computed per state, not per task

   more expensive than other landmark heuristics
- Accuracy can be improved (at further computational expense) using hitting sets (Bonet & Helmert, 2010)
  - 1. Run procedure multiple times to get different (possibly overlapping) disjunctive landmarks
  - 2. Compute a set of actions that hits each disjunction
  - ~ Optimal hitting set NP-hard, approximate

### Heuristics for Domain-Independent Planning 6. Abstraction Heuristics

Emil Keyder Silvia Richter

Based on slides by Carmel Domshlak and Malte Helmert

ICAPS 2011 Summer School on Automated Planning and Scheduling

June 2011

Emil Keyder, Silvia Richter

Heuristics: 6. Abstraction Heuristics

June 2011 1 / 36

Introduction Transition Systems

Motivation

Like delete-relaxation heuristics, abstraction heuristics are derived from a simplification of the task.

But here, we simplify the search space directly, rather than via the operators.

### Introduction

Transition Systems Abstractions

### Pattern Database Heuristics

Introduction Examples Multiple & Additive Patterns Finding Good Patterns

### Merge-and-Shrink Heuristics

Introduction			
Merging			
Shrinking			
Generic Algorithm			
Theoretical Propert	ies		
Structural Pattern Heu	ristics		
Emil Keyder, Silvia Richter	Heuristics: 6. Abstraction Heuristics	June 2011	2 / 36





### Transition Systems of SAS<sup>+</sup> Planning Tasks

Definition (transition system of an SAS<sup>+</sup> planning task) Let  $\Pi = \langle V, s_0, G, O, cost \rangle$  be an SAS<sup>+</sup> planning task. The transition system of  $\Pi$ , in symbols  $\mathcal{T}(\Pi)$ , is the transition system  $\mathcal{T}(\Pi) = \langle S', L', T', I', G' \rangle$ , where

• S' is the set of states over V,

• 
$$L' = O$$
,

• 
$$T' = \{ \langle s', o', t' \rangle \in S' \times L' \times S' \mid app(s', o') = t' \},$$

• 
$$I' = s_0$$
, and

$$\blacktriangleright G' = \{s' \in S' \mid s' \models G\}.$$

Emil Keyder, Silvia Richter

Heuristics: 6. Abstraction Heuristics

June 2011

5 / 36



Introduction Transition Systems		
Example Task: One Package, Two Trucks		
Example (one package, two trucks)		
Consider a Logistics task with		
► Two locations: L, R; two trucks: A, B; one package p		
▶ Package can be at locations or in trucks: D <sub>p</sub> = {L, R, A, Trucks can be at the locations: D <sub>tA</sub> = D <sub>tB</sub> = {L, R}	B}	
Init state: package at L, both trucks at R		
► Goal: package at R		
<ul> <li>Operators for pickup, drop and move</li> </ul>		
<ul> <li>pickup<sub>i,j</sub>: pickup the package with truck <i>i</i> at location <i>j</i></li> <li>drop<sub>i,j</sub> analogously</li> </ul>		
• move $i, j, j'$ : move truck <i>i</i> from location <i>j</i> to $j'$		
Emil Keyder, Silvia Richter Heuristics: 6. Abstraction Heuristics	June 2011	6 / 36

Introduction Abstractions

### Abstracting a Transition System

Abstracting a transition system means dropping some distinctions between states, while preserving the transition behaviour as much as possible.

An abstraction of a transition system *T* is defined by an abstraction mapping α that defines which states of *T* should be distinguished and which ones should not.

 $\rightsquigarrow$  *s*, *t* not distinguished if  $\alpha(s) = \alpha(t)$ 

Emil Keyder, Silvia Richter

From *T* and *α*, we compute an abstract transition system *T'* which is similar to *T*, but smaller.

The abstract goal distances (goal distances in  $\mathcal{T}'$ ) are used as heuristic estimates for goal distances in  $\mathcal{T}$ :

$$h^{\alpha}(s,T) = h^*(\alpha(s),T')$$

Heuristics: 6. Abstraction Heuristics







- How to come up with a suitable abstraction mapping  $\alpha$ ?
- Conflicting goals for abstractions:
  - we want to obtain an informative heuristic, but
  - want it to be efficiently computable (both abstract state α(s) and h\*(α(s)))
- Combinations of several abstractions possible





June 2011

14 / 36

Definition (projections)

 $P \subseteq V$ , and let S' be the set of states over P.

We call *P* the pattern of the projection  $\pi_P$ .

(with  $s|_P(v) := s(v)$  for all  $v \in P$ ).

they agree on all variables in P.

The projection  $\pi_P: S \to S'$  is defined as  $\pi_P(s) := s|_P$ 

Projections

PDBs

PDBs: abstraction mappings are projections to certain state variables.

Let  $\Pi$  be an SAS<sup>+</sup> planning task with variable set V and state set S. Let

In other words,  $\pi_P$  maps two states  $s_1$  and  $s_2$  to the same abstract state iff

Introduction



PDBs Multiple PDBs

Pattern Collections

Space requirements for pattern databases grow exponentially with number of state variables in pattern.

 $\rightsquigarrow$  Not practical for larger planning tasks.

Instead use collections of smaller patterns.

- Max of PDB heuristics is admissible (general property)
- Preferable to use sum when possible



### Criterion for Additive Patterns

Simple criterion for admissible additivity: Count cost of each action in at most one heuristic.

PDBs

Multiple PDBs

 $\rightsquigarrow$  Can add heuristic estimates from two patterns if no operator affects variables in both

Can find maximal additive subsets of a set of patterns

For a collection of patterns  $\mathcal P,$  the max of these sums is the canonical heuristic function for  $\mathcal P$ 

 $\rightarrow$  Most informative admissible heuristic we can derive from  ${\cal P}$ 

Can do better with cost partitioning

### How Do We Come Up with Good Patterns?

- The biggest practical problem when applying pattern databases to planning is to find a good pattern collection in a domain-independent way.
- Most promising approach: search in the space of possible pattern collections (prior to actual planning)

### Algorithm of Haslum, Helmert, Bonet, Botea & Koenig (2007)

- Hill-climbing search in space of possible pattern collections
- Start from all singleton patterns  $\{\{v\} \mid v \in V\}$
- Neighbors of collection P: P ∪ {P ∪ {v}}
   for some P ∈ P and v ∉ P
- Evaluation of a pattern collection  $\mathcal{P}$ :
  - Randomly sample heuristic for some states
  - ► Use search effort formula (Korf, Reid & Edelkamp, 2001)

Heuristics: 6. Abstraction Heuristics

Merge-and-Shrink Heuristics Introduction

### Merge-and-Shrink Abstractions: Main Idea

### Main idea of M&S abstractions

(due to Dräger, Finkbeiner & Podelski, 2006):

Instead of perfectly reflecting a few state variables, reflect all state variables, but in a potentially lossy way.

### Do this with a sequence of

- 1. Merge steps add a new variable to the abstraction
- 2. Shrink steps reduce abstraction size by fusing abstract states

### Beyond Pattern Databases

Major limitation of PDBs: patterns must be small

Consider Logistics example with N trucks, M locations (still one package):

- ▶ If package  $\notin P$ , h = 0
- ▶ If any truck  $\notin P$ ,  $h \leq 2$

P must include all variables to improve over  $P' = \{ package \}$ 

Merge-and-shrink (M&S) abstractions are a proper generalization of pattern databases.

- They can do everything that pattern databases can do (modulo polynomial extra effort)
- They can do some things that pattern databases cannot

Emil Keyder, Silvia Richter

Heuristics: 6. Abstraction Heuristics

June 2011 22 / 36

Merge-and-Shrink Heuristics Merging

### M&S Abstractions: Key Insights

### Key insights:

- 1. Information of two transition systems  $\mathcal{A}$  and  $\mathcal{A}'$  can be combined (without loss) by a simple graph-theoretic operation, synchronized product  $\mathcal{A} \otimes \mathcal{A}'$
- 2. The complete state space of a planning task can be recovered using only atomic projections:

 $\bigotimes_{\mathbf{v}\in\mathcal{V}}\pi_{\mathbf{v}} \text{ is isomorphic to } \pi_{\mathcal{V}}.$ 

 $\rightsquigarrow$  build fine-grained abstractions from coarse ones

June 2011



26 / 36

### Merge-and-Shrink Heuristics Shrinking

### M&S Abstractions: Key Insights

### Key insights:

- 1. Information of two abstractions  $\mathcal{A}$  and  $\mathcal{A}'$  of the same transition system can be combined by a simple graph-theoretic operation (synchronized product  $\mathcal{A} \otimes \mathcal{A}'$ ).
- 2. The complete state space of a planning task can be recovered using only atomic projections:

$$\bigotimes_{\nu \in \mathcal{V}} \pi_{\nu} \quad \text{is isomorphic to} \quad \pi_{\mathcal{V}}.$$

- $\rightsquigarrow$  build fine-grained abstractions from coarse ones
- 3. When intermediate results become too big, we can shrink them by fusing some abstract states.

Emil Keyder, Silvia Richter

Heuristics: 6. Abstraction Heuristics

June 2011 29 / 36







### Merge-and-Shrink Heuristics Generic Algorithm

### Computing M&S Abstractions

### Generic abstraction computation algorithm

abs := all atomic projections  $\pi_v$  ( $v \in V$ ). while abs contains more than one abstraction: select  $A_1$ ,  $A_2$  from abs shrink  $A_1$  and/or  $A_2$  until  $size(A_1) \cdot size(A_2) \leq N$ abs := abs \ { $A_1, A_2$ }  $\cup$  { $A_1 \otimes A_2$ } return the remaining abstraction

*N*: parameter bounding number of abstract states

### Questions for practical implementation:

- ► Which abstractions to select? ~> merging strategy
- How to shrink an abstraction?  $\sim$  shrinking strategy
- ► How to choose N?

### Emil Keyder, Silvia Richter

Heuristics: 6. Abstraction Heuristics

Structural Pattern Heuristics

### Limitation of Explicit Abstractions

No tricks: abstract spaces are searched exhaustively

- $\rightsquigarrow$  must keep abstract spaces of fixed size
- $\sim$  (often) price in heuristic accuracy in long-run





### Structural Pattern Heuristics

### Structural Abstraction Heuristics: Main Idea

Instead of perfectly reflecting a few state variables, reflect many (up to  $\Theta(|V|)$ ) state variables, but

guarantee abstract space can be searched (implicitly) in poly-time

### How

Abstracting  $\Pi$  by an instance of a tractable fragment of cost-optimal planning

 $\rightsquigarrow$  Fork Decomposition (Katz & Domshlak, 2008): decompose causal graph of task into fragments with single root/sink variable (and use further tricks)

Details: Carmel Domshlak's lecture in this summer school

June 2011

### Heuristics for Domain-Independent Planning 7. Final Comments Emil Keyder Silvia Richter Based partially on slides by Carmel Domshlak and Malte Helmert ICAPS 2011 Summer School on Automated Planning and Scheduling June 2011



- We have seen a number of heuristics in this tutorial, for satisficing and optimal planning
- ► Now the big question is:

### Which ones work best?

And what's there beyond heuristics to improve planning-by-search?

Satisficing Planning Beyond Heuristics			
Optimal Planning			
Emil Keyder, Silvia Richter	Heuristics: 7. Final Comments	June 2011	2 / 7

### Satisficing vs. Optimal Planning

Satisficing and optimal planning are much more different from each other than it appears at a superficial glance.

Optimal planning entails proving that there is no better solution

Satisficing Planning

- As a consequence, A\*-based planning requires exponential effort in most domains even with almost perfect heuristics (Helmert & Röger, 2008).
- ▶ Not so for satisficing planning, where just finding a solution is enough

### Satisficing Planning Beyond Heuristics

### Satisficing Planning: Beyond Heuristics

- Because satisficing planners do not need to prove optimality, they can use drastic search control strategies
- These often make a larger difference in performance than the choice of heuristic function (Richter & Helmert, 2009)

### Examples:

- helpful actions (FF: Hoffmann & Nebel, 2001)
- relaxed plan macros (YAHSP: Vidal, 2004)
- preferred operators (Fast Downward, LAMA: Helmert, 2006)
- alternating multi-queue search (Fast Downward, LAMA: Helmert & Röger, 2010)

### Emil Keyder, Silvia Richter

Heuristics: 7. Final Comments

Optimal Planning

### **Optimal Planning: Conclusion**

- Landmark heuristics like h<sup>LA</sup> and h<sup>LM-cut</sup> shown to outperform additive h<sup>max</sup> and abstraction heuristics like PDBs and M&S (under typical competition conditions)
- Theoretical results show M&S has the power to be more informative than PDBs and landmark heuristics, while additive h<sup>max</sup> ~ landmarks (Helmert & Domshlak, 2009)
- But worth keeping in mind that higher accuracy does not necessarily pay off when time is limited

### Satisficing Planning: Conclusion

- Satisficing heuristics with good coverage (given limited time) are e.g. the FF heuristic and the context-enhanced additive heuristic, whereas the additive heuristic and inadmissible landmark-counting (by themselves) are less strong

in particular when planning with action costs

E.g. cost-insensitive variant of FF heuristic dominates cost-sensitive one in terms of the IPC 2008 criterion (Richter & Westphal, 2010)

Emil Keyder, Silvia Richter

Heuristics: 7. Final Comments

June 2011 6 / 7

June 2011



ICAPS 2009 Summer School. Focus on admissible heuristics, basis of several parts of this tutorial.

### Emil Keyder and Blai Bonet. Heuristics for Classical Planning (With Costs). ICAPS 2009 tutorial. Emphasis on inadmissible heuristics based on delete relaxation.

Erez Karpas and Silvia Richter. Landmarks - Definitions, Discovery Methods and Uses. ICAPS 2010 tutorial. Basis of the first landmark part of this tutorial.

Jörg Hoffmann and Bernhard Nebel. The FF Planning System: Fast Plan Generation Through Heuristic Search. JAIR 14:253-302, 2001. Introduces FE and the FE heuristic.

Artificial Intelligence 129(1):5-33, 2001.

A foundational paper on planning as heuristic search.

Introduces max heuristic and additive heuristic.



Improving the accuracy of the landmark cut heuristic. Heuristics: References

June 2011

### References: Critical Path Patrik Haslum and Hector Geffner Admissible Heuristics for Optimal Planning. Proc. AIPS 2000, pp. 140-149, 2000. Introduces critical path heuristics. Patrik Haslum $h^m(P) = h^1(P^m)$ : Alternative Characterisations of the Generalisation From $h^{max}$ to $h^m$ . Proc. ICAPS 2009, pp. 354-357, 2009. Introduces $P^m$ compilation. Emil Keyder, Silvia Richter Heuristics: References June 2011 9 / 21



June 2011

11 / 21

Heuristics: References

Emil Keyder, Silvia Richter

Ref	Ferences: Pattern	Databases		
	Stefan Edelkamp. Planning with Patter <i>Proc. ECP 2001</i> , pp. First paper on plann	n Databases. 13–24, 2001. ing with pattern databases.		
	Stefan Edelkamp. Symbolic Pattern Da <i>Proc. AIPS 2002</i> , pp Uses BDDs to store	atabases in Heuristic Search Pla b. 274–283, 2002. pattern databases more compa	anning. Ictly.	
E	Emil Keyder, Silvia Richter	Heuristics: References	June 2011	10

### References: Pattern Databases (ctd.)

Emil Keyder, Silvia Richter

- Patrik Haslum, Malte Helmert, Adi Botea, Blai Bonet and Sven Koenig.
   Domain-Independent Construction of Pattern Database Heuristics for Cost-Optimal Planning.
   Proc. AAAI 2007, pp. 1007–1012, 2007.
   Introduces canonical heuristic for pattern collections.
   Search-based pattern selection based on Korf, Reid & Edelkamp's theory.
   Marcel Ball and Robert C. Holte.
  - Marcel Ball and Robert C. Holte.
     The Compression Power of Symbolic Pattern Databases.
     *Proc. ICAPS 2008*, pp. 2–11, 2008.
     Detailed empirical analysis showing benefits of symbolic BDDs over explicit-state BDDs.

Heuristics: References

### References: Merge & Shrink Klaus Dräger, Bernd Finkbeiner and Andreas Podelski. Directed Model Checking with Distance-Preserving Abstractions. Proc. SPIN 2006, pp. 19–34, 2006. Introduces merge-and-shrink abstractions (for model-checking). Malte Helmert, Patrik Haslum and Jörg Hoffmann. Flexible Abstraction Heuristics for Optimal Sequential Planning. Proc. ICAPS 2007, pp. 176–183, 2007. Introduces merge-and-shrink abstractions for planning.

Heuristics: References

June 2011

13 / 21

Emil Keyder, Silvia Richter







### References: Heuristic Performance

- Malte Helmert and Gabriele Röger.
   How Good is Almost Perfect?
   Proc. AAAI 2008, pp. 944–949, 2008.
   Shows that optimal planning is hard even with almost perfect heuristics differing from h\* by an additive constant.
- Malte Helmert and Robert Mattmüller. Accuracy of Admissible Heuristic Functions in Selected Planning Domains.

### *Proc. AAAI 2008*, pp. 938–943, 2008.

Introduces asymptotic accuracy of admissible heuristics.

Emil Keyder, Silvia Richter

Heuristics: References

June 2011 17 / 21

### References: Search Enhancements (ctd.)

Silvia Richter and Malte Helmert.
 Preferred Operators and Deferred Evaluation in Satisficing Planning.
 *Proc. ICAPS 2009*, pp. 273–280, 2009.
 Empirical evaluation of various strategies for preferred operators.

### Gabriele Röger and Malte Helmert.

The More, the Merrier: Combining Heuristic Estimators for Satisficing Planning.

*Proc. ICAPS 2010*, pp. 246–249, 2010.

Empirical evaluation of various strategies for combining several heuristics.

### References: Search Enhancements

Jörg Hoffmann and Bernhard Nebel. The FF Planning System: Fast Plan Generation Through Heuristic Search. JAIR 14:253-302. 2001. Introduces relaxed plans and helpful action pruning. Vincent Vidal. A Lookahead Strategy for Heuristic Search Planning. In Proc. ICAPS 2004, pp. 150-159, 2004. Takes helpful actions further with relaxed plan macros. Malte Helmert. The Fast Downward Planning System. JAIR 26:191-246, 2006. Introduces preferred operators, alternating multi-queue search, and delayed evaluation for systematic best-first search algorithms. Emil Keyder, Silvia Richter Heuristics: References June 2011 18 / 21

### References: Heuristic Analysis & Comparison (ctd.)

### Michael Katz and Carmel Domshlak. Optimal Additive Composition of Abstraction-based Admissible Heuristics. *Proc. ICAPS 2008*, pp. 174–181, 2008. Introduces action cost partitioning, and proves tractability of optimal partitions for (numerous types of) abstractions. Malte Helmert and Carmel Domshlak. Landmarks, Critical Paths and Abstractions: What's the Difference Anyway? *Proc. ICAPS 2009*, pp. 162–169, 2009. Uses cost partitioning to prove formal compilability results between various classes of heuristics.

References: Heuristi	c Analysis & Compari	son	
Jörg Hoffmann. Where Ignoring Delet <i>Proc. ICAPS 2011</i> , 2 Automated analysis o domains.	e Lists Works, Part II: Caus 011. f delete-relaxation heuristics	al Graphs s on different	
Emil Keyder, Silvia Richter	Heuristics: References	June 2011 :	21 / 21